Positronium Decay Experiment Laboratory Manual - Revised

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In this experiment, you will observe the creation of free positrons through ^{22}Na β^+ decay and the subsequent creation of positronium. Using a variety of advanced instrumentation, you will then explore the decay properties of positronium. This relatively open-ended lab will allow you to further perform any of your own experiments with positronium decay.

I. INTRODUCTION

Positronium is an onium type exotic atom composed of an electron (e^-)bound with its antiparticle, the positron (e^+). This unique combination of provides an invaluable tool to study quantum electrodynamics (QED), as the pair of leptons allows the isolation of leptonic properties and photon fields without the much more massive nucleus.

The antielectron, or positron, was first conceptualised by Paul Dirac in 1927. His famed Dirac equation postulated that electrons were allowed both positive and negative energy as solutions to the equation. Experimentally, the positive energy solution was verified via the electron, but mathematically, the negative energy solution could be just as valid.[1] In 1931, building off of the controversy created by the negative energy solution to the Dirac equation, Dirac theorised a new particle called an "antielectron" with the equivalent mass, angular momentum, and wave properties as the electron, but with positive charge and thus negative energy.[2] In 1929, Dmitri Skobeltsyn first observed the positron while trying to detect gamma radiation in cosmic rays in a Wilson cloud chamber by noticing that there seemed to be an electron-like particle that curved the opposite way in an external magnetic field.[3] In 1932, Carl Anderson repeated this experiment with the intention of exploring this antielectron, the first evidence of antimatter, for which he won the 1936 Nobel Prize in Physics.[4,5]

This junior lab experiment is very involved and there is much to learn from performing this experiment. This experiment is relevant to modern physics as it still a common topic of research. Positronium was first observed by Martin Deutsch at MIT in 1951. Based on his work, positronium has been used extensively in research.[6] The interactions of electrons and positrons can be described by their fundamental leptonic properties and photon fields, thus making positronium a common topic of research for quantum electrodynamics. Detection and measurement of excited states of positronium is currently being researched in hopes of better understanding QED and the electro-weak force. Positronium and positron interactions with larger complex molecules have also been topics of research in radiation chemistry[6]. Positronium has been shown to form bonds with larger molecules to form complex compounds, such as PsH, or positronium hydride. These interactions have been invaluable to radiation chemistry research. The study of positronium behaviour has been vital to research in charge conjugation, parity, and time-reversal (CPT) symmetry and invariance. Further modern physics research continues as researchers search for forbidden decay modes of positronium in an attempt to test time-reversal, parity and charge conjugation invariance[6]. In addition, this experiment will help to develop your laboratory techniques for data acquisition and analysis. You will be able to operate and learn about sophisticated and intricate devices used for detection and measurements. There is a great deal of complicated physics to be learned from performing this experiment. Likewise you will observe the impressive physical phenomenon of matter-antimatter annihilation. This experiment is excellent preparation for more complicated and involved labs to come.

In this experiment, you will use the β^+ decay of the unstable sodium isotope ^{22}Na as a positronium source. Using thallium(Tl)-doped sodium iodide(NaI) scintillators, you will able to observe the gamma rays associated with the creation and annihilation of the positron and positronium in the ^{22}Na decay, which you will then use to observe the lifetime time of positronium events.

II. THEORETICAL BACKGROUND

To fully understand the experiment it is important to understand how the physics of the experiment translate into the data which you will analyze to measure the decay rate of orthopositronium. In this section we will cover the theory behind the detection of orthopositronium. Although this lab manual will briefly cover the physics involved in this experiment, from positronium formation to detection and counting mechanisms, you should conduct further research for clarification and detail.

A. β^+ Decay

The unstable isotope ^{22}Na undergoes what is known as β^+, or *positron emission*, decay, in which a proton decays into a neutron, releasing a positron and neutrino

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(Figure 1). ^{22}Na decays into a 1.275 MeV excited state of ^{22}Ne, a positron, and a neutrino. The ^{22}Ne quickly (instaneously on our experimental timescale) decays to its stable ground state, emitting a "high energy" 1.275 MeV gamma ray in the process.[7] As noted in the Experimental Procedure, this gamma photon detection is used as the start signal for our timer mechanism, alerting us that a free positron has been formed.

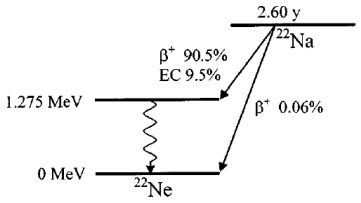

Figure 1. The β^+ decay of ^{22}Na.[7]

The binding energy of the daughter atom must exceed that of the original nucleus by at least $m_n c^2 - m_p c^2 + m_e c^2$, or 2.511 MeV, in order to observe β^+ decay. (Note that m_n is neutron mass, m_p is proton mass, and m_e is electron mass.) The binding energy of ^{22}Na, 174.15 MeV, subtracted from the binding energy of ^{22}Ne, 177.78 MeV, yields a binding energy difference of 3.63 MeV, which exceeds the minimum binding energy difference and allows for ^{22}Na β^+ decay.[10] Additionally, as a positron is emitted from the decay of the original nucleus, an electron must be shed from the outer orbital of the daughter atom in order to balance the charge. This means that at least two electron masses are shed in the process of β^+ decay and the resulting daughter atom must be at least two electron masses (1.022 MeV) lighter than the original atom.[9] These two energy-mass conditions must be met for β^+ decay to occur. [8]

B. Positronium Formation

The positron resulting from the β^+ decay is quite energetic. According to *Ore Gap* theory, certain energy conditions must be fulfilled in order for an electron and positron to bind into positronium. In order to maintain a bound state with the electron and then undergo annihilation, the energy of positronium after formation, E_{Ps}, must be less than the dissociation energy of positronium, I_{Ps}. Thus, as $E_{Ps} < I_{Ps}$, the emitted positron kinetic energy, E_{e^+}, must be within limits in order for the positron to bind with an electron and produce positronium. To facilitate this, the high energy positron undergoes inelastic collisions in the gaseous medium, in our case, nitrogen, until it slows to an energy sufficient for electron capture. Neglecting kinetic energy, the energy of the resulting positronium is then E_{Ps} such that $E_{Ps} = E_{e^+} - I_M + I_{Ps}$, where I_M is the ionization energy of the gas molecule. This relationship demonstrates that the energy of the free positron must be less than the ionization energy, i.e. $E_{e^+} < I_M$. However, the energy must be sufficient as to allow the positron to capture an atomic electron from its host, thus $E_{e^+} > I_M - I_{Ps}$ Positrons with energies below this threshold are unable to free an atomic electron in order to form a bound positronium atom and proceed to elastically interact with the medium until eventually annihilating with a free electron.[7]

C. Positronium Properties

There are two possible bound states for ground-state positronium after it has formed. These states are related to the relative spin states of the positron and electron in the positronium atom. *Orthopositronium* is a triplet state of positronium characterised by parallel spin states between the positron and a total spin of $s = 1$. Conversely, *parapositronium* is a singlet state of positronium characterised by antiparallel spins between the positron and electron and a total spin of $s = 0$.[7] Significant overlap in the positron and electron wave functions lead the positron to eventually annihilate such that all of the positronium energy is converted into gamma rays. The observation of these gamma rays, as described in the experimental procedure, will be our stop signal for the decay time measurements. The number of gamma rays emitted from annihilation corresponds to the total spin s via the charge conjugation selection rule:

$$(-1)^{l+s} = (-1)^n \tag{1}$$

where l is the relative orbital angular momentum, and n is the number of gamma rays emitted through the annihilation process. l must be zero in the ground state of positronium, therefore the number of gamma photons released depends exclusively on the total spin number. Thus, orthopositronium must always decay into an odd number of photons while parapositronium must always decay into an even number of photons. As the release of a single gamma photon would violate conservation of momentum and energy, the most likely decay scenario for orthopositronium becomes three gamma photons, while parapositronium decays most probably into two gamma rays of energy 0.511 MeV each which travel in opposite directions, reflecting the mass-energy of the positron and electron pair. The orthopositronium decay also divides the total positronium energy among the the emitted photons; the photons emitted through orthopositronium decay exhibit a continuous energy distribution. The process of positronium decay that leads to multiple photon emission is reflected in the relatively long lifetimes of bound positron-electron pairs, with orthopositronium having a longer lifetime than parapositronium. Spin-exchange interactions with certain surrounding gaseous media with two available spin states (such as oxygen) can, however, cause spin changes in the positron or electron in the bound pair, thus converting orthopositronium into parapositronium in a process referred to as "pick-off".

The resulting parapositronium decays quicker than the orthopositronium, reducing the observed lifetime of the orthopositronium.[1] This pick-off is important to note as it may be a significant factor in the deviation of your lifetime measurements from expected values.

D. Positronium Decay

The random decay of positronium can be modeled with a differential equation relating the decay rate $-dN/dt$ as a function of the population of the positronium as a function of time:

$$\frac{-dN}{dt} = \lambda N(t) \tag{2}$$

where $N(t)$ is the population of the positronium as a function of time and λ is the decay constant of the positronium in the medium, given in units of time^{-1}.[10] The decay constant represents the rate of the exponential decay of the positronium, an intrinsic property of the positronium in the medium. The solution to Equation 2 can then be given as:

$$N(t) = N_0 e^{-\lambda t} \tag{3}$$

where N_0 is the initial population of the positronium atoms in a sample.[10] This exponential function was later fitted on a histogram of measured decay times in order to solve for the decay constant. In our analysis, N_0 was arbitrary and inconsequential, as our ^{22}Na sample allowed us a steady rate of positronium creation. The half-life of the positronium $t_{1/2}$ can also be related to the decay constant as:

$$e^{-\lambda t_{1/2}} = 1/2 \tag{4}$$

[2]

E. Experimental Apparatus

Here we will explain each of the component instruments required for this experiment.

1. Sodium Iodine (NaI) Scintillators

For this experiment, you will be using a thallium-doped sodium iodide scintillator. This device effectively converts an emitted gamma ray into visible light which is then detected with a photomultiplier tube via the photoelectric effect. A gamma ray enters the crystal and excites a single electron from the valence band into the conduction band [11]. The electron traverses through the lattice, colliding and dispersing the total energy until the excitation energy of each electron matches the activation band gap of the thallium impurity center. The excited thallium in the sodium iodide crystal then becomes a local center for electron—electron-hole interactions where various electrons energy transitions occur to emit light. These process of radiation production via the activated impurity center is described in Figure 2. The intensity of light detected by the photomultiplier tube is linearly proportional to the energy lost by the initially excited electron. This scintillator and photomultiplier tube allow the gamma ray to be detected and converted into a voltage pulse proportional to the energy of the gamma ray [12]. You will use these instruments to detect the gamma rays emitted from both the β^+ decay and positron-electron annihilation.

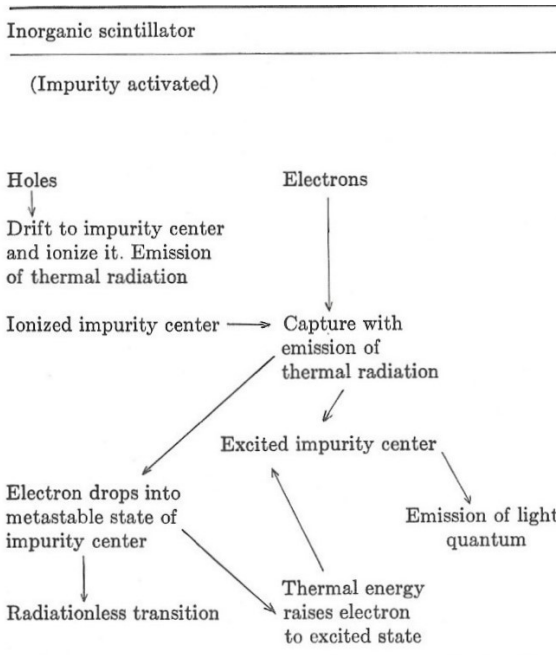

Figure 2. Processes Leading to Emission of Light in NaI Scintillator (taken from [12])

2. Time-to-Amplitude Converter (TAC)

The discriminator will produce a digital voltage pulse for each analog voltage pulse it detects above a set potential. The threshold potential you will set will be explained in the experiment setup. The pulses from the discriminator will be sent to the time to amplitude converter. The start on this device will be triggered by the pulse generated from the ^{22}Ne relaxation photon. It will then begin to apply a current through a capacitor which will accumulate charge. This current will stop at the signal of the voltage pulse produced by the gamma ray created during positron-electron annihilation. The charge accumulated on the capacitor will be used to translate into a voltage pulse proportional to the time be-

tween gamma rays. These pulses will be counted through the multichannel analyzer (MCA) by a computer with the GammaVision software to produce a histogram of positronium lifetimes.

3. Time Calibrator

The Time Calibrator is used to calibrate the amplitude signal from the TAC. The Time Calibrator works by outputting time-delayed pulses on a set periodic interval, allowing us to measure the amplitude from the TAC and associate it with a precise time value. The Calibrator has two adjustable functions. The "Period" setting sets the delay interval between start and subsequent stop pulses. The range sets the total range for the stop pulses. For example, if the period is set to 0.08μs and the range is set to 1.28μs, the Time Calibrator will continuously output a start signal followed by a stop signal delayed by random multiple of the period, 0.08μs, up to the range value of 1.28μs; i.e. a start signal may be followed by stop pulses delayed by either 0.08, 0.16, 0.24, ..., or 1.28μs. If you then plot a histogram of each of the subsequent TAC signals as a function of potential amplitude in a program such as GammaVision, you would see discrete peaks in the TAC amplitude, the spacing of which would correspond exactly to the period set on the Time Calibrator. By comparing the proportionality of the TAC signal amplitude and the spacing distance, you can then calculate the exact relationship between the TAC signal amplitude and start-stop signal delay. Thus, the TAC voltage amplitude can later be expressed in units of time.

4. Oscilloscope Usage

This experiment requires extensive use of the oscilloscope. Please be sure you understand the mechanics of an oscilloscope before preforming the experiment. You will be utilizing a total of 5 oscilloscope channels when preforming this experiment. The first four oscilloscope channels will be used to view the voltage pulses generated by the photomultiplier tubes as well as the pulses that are produced by the discriminator. You will set the oscilloscope to trigger off of the discriminator signals so that only the discriminator-selected voltage pulses are detected. With proper calibration, these oscilloscopes will then effectively display voltage pulses from the photomultiplier tube that are proportional in energy to the gamma rays emitted during β^+ decay and annihilation. The last oscilloscope will display the pulse produced by the TAC. The amplitude of this voltage pulse will be proportional to the lifetime of the positronium atom. This pulse also marks the formation and annihilation of positronium.

III. EXPERIMENTAL PROCEDURE

We will be using the decay of a small sample of ^{22}Na to provide the constant source of positrons for our experiment. The sample will be surrounded by two thallium-doped sodium iodide scintillators with photomultiplier tubes (PMTs), which will be used to detect the gamma photons corresponding to the creation and destruction of the positronium. Using a variety of instrumentation, we will measure the time between the constructive and destructive observations of the scintillators to determine the lifespan and decay rate of both parapositronium and orthopositronium.

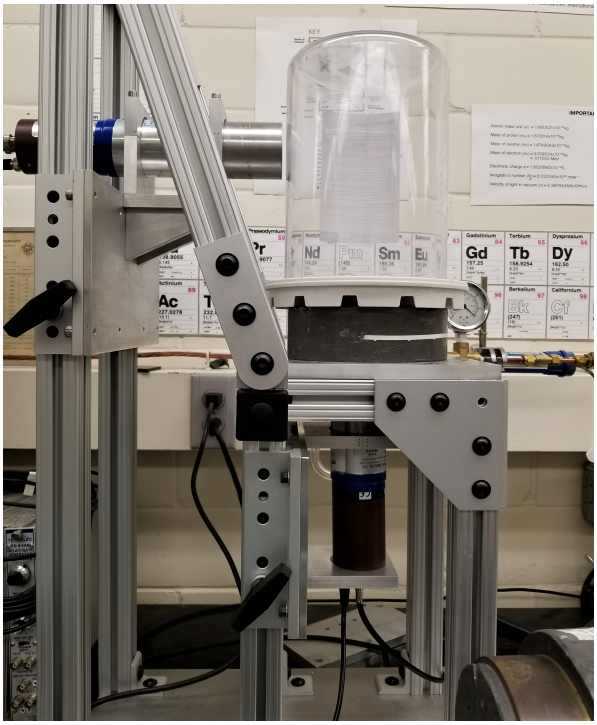

Figure 3. A photograph of our setup of the scintillating tubes. The sample was placed in the bottom of the vacuum jar over the center of the lead ring.

First, you will want to set up the experimental apparatus. Figure 4 provides a visual block diagram of the entire setup for reference. We found "following the signal" to be a logical form of setting up the experiment in order to fully understand each component. We know that the decay of ^{22}Na results in the production of a very high-energy gamma photon. We can reasonably assume that the release of this photon coincides with the release of a positron and the subsequent formation of positronium. There may be some delay in the formation of the positron-electron pair after the initial release of the positron, but this time gap would likely be much smaller than the uncertainty in our measurements and thus can be ignored. It would be wise to note this discrepancy, however, in the errors and uncertainties section in your report.

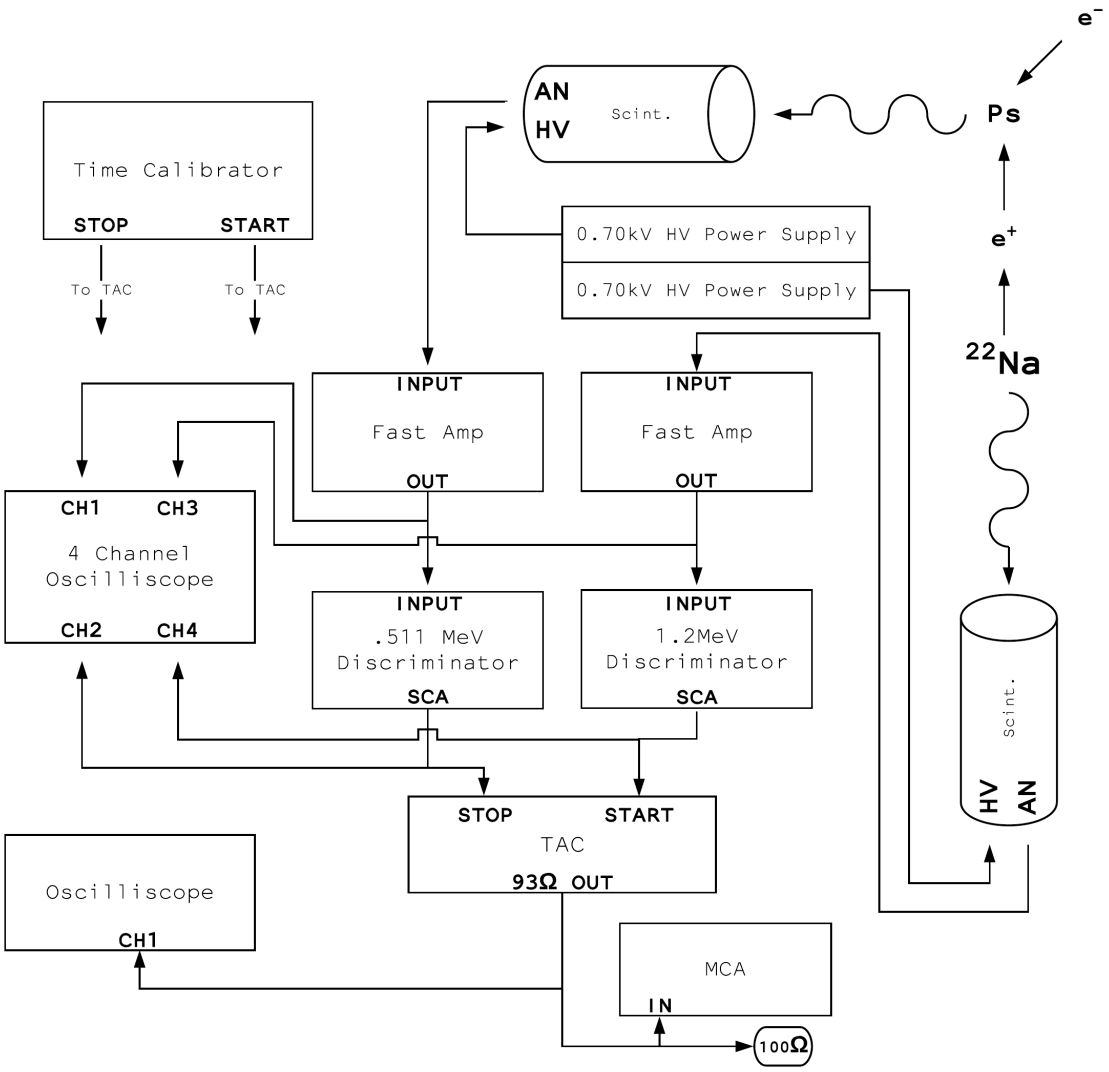

Figure 4. A block diagram of the experimental setup.

The lab has two scintillating tubes available for use. They will likely be mounted in the experimental apparatus as pictured in 3. The ^{22}Na sample is placed over the longitudinal axis of the vertically mounted scintillator, in the base of the vacuum jar. The "HV" (High Voltage) port on the back of the scintillator should be connected via the special high-voltage cable to the output on one of the high-voltage power supplies, set to 0.7 kV.

The anode ("AN") on the scintillator should then be connected to one of the input channels of the **Fast Amplifier**. The amplifier serves to boost the voltage of the signal coming from the PMT, which, in our experiment, was far too low to allow us to effectively set the discriminator threshold voltage in later steps. Depending on the scintillator you use, you may or may not require amplification of the signal, so bear that in mind when making adjustments to the final setup.

You will want to connect a coaxial "T" junction to the output of the fast amp. This will allow you to directly monitor the signal from the PMT (post-amplification) by connecting one of the outputs on the T junction to channel 1 of the four channel oscilloscope (refer to 4). The other end of the T junction should be connected to the input of one of the channels on the **Discriminator**. Depending on the discriminator used, you may need a special adapter to connect the coaxial cable to the discriminator, so be sure to ask your instructor for the proper connector. Similarly, connect the output on the discriminator to another coaxial T junction. This time, the T junction should be connected to channel 2 of the same oscilloscope and "START" input on the **Time to Amplitude Converter (TAC)**.

At this point, we will calibrate the discriminator channel to detect only the initial high-energy gamma ray emit-

ted by the ^{22}Ne relaxation following the ^{22}Na decay. Go ahead and turn on the HV power supply connected to the scintillator. Set it to output around 0.7 kV, although this value may change with your specific scintillator. You should be able to see some sort of signal appear on the oscilloscope. We will configure the oscilloscope shortly. Once you have the power supply set, you will return to the discriminator. Recall that the voltage from the PMT is directly proportional to the energy of the incident photon. Also remember that the discriminator works by emitting a digital pulse every time the input signal rises above the set threshold. We should be able to see this interaction on the oscilloscope.

Set the scope to trigger off of channel 1, the signal directly from the PMT. Set the trigger level to a fairly low amplitude, close to the zero line, while avoiding the signal noise. If you do not see the signals after adjusting the positioning, scaling, and trigger, you may want to adjust the coupling on the scope and the DC offset on the instrumentation.

The oscilloscope display should now look similar to Figure 5. Play with the persist setting until the display looks to your liking. It should be apparent that there seem to be two very common pulse height bands on the oscilloscope display. These correspond to the two most prominent energy peaks in the ^{22}Na decay spectrum, conveniently the ^{22}Ne relaxation peak and the positron-electron annihilation peak. Given that the pulse amplitude is proportional to the incident gamma energy, it should be apparent that the higher amplitude band corresponds to the ^{22}Ne relaxation photon while the lower band corresponds to the positronium annihilation photon.

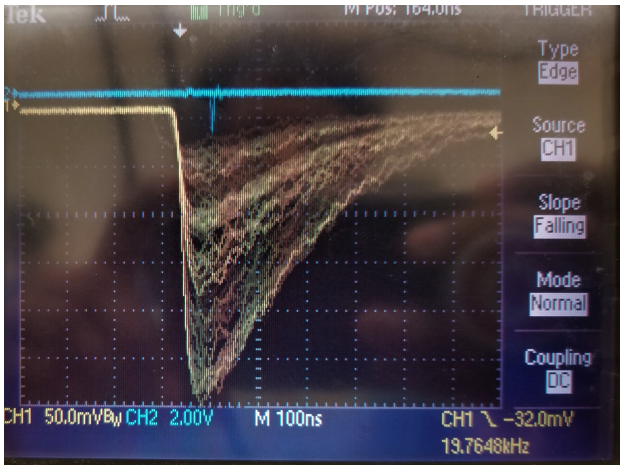

Figure 5. The oscilloscope display showing the scintillator pulses in yellow. Note the prominent bands corresponding two the two prominent ^{22}Na peaks.

Now set the scope to trigger off of channel 2, the signal directly from the discriminator. Set the trigger level appropriately and you should be able to see both the digital pulse and the initial signal from the PMT superimposed on the scope display.

We are looking for the high energy photon released by the ^{22}Na decay, so by setting the discriminator threshold very high initially, we can slowly dial it backwards until we only see the high amplitude peak corresponding to the high-energy photon. Go ahead and adjust the discriminator in this fashion. You will want to play with the window size on the discriminator to isolate the band as well. The discriminator channel should now be properly configured.

We will repeat most of these steps now for the second scintillator. Once again, connect the second scintillator tube (mounted horizontally as in Figure 3) to its own HV power supply. The tube should be placed in a manner to where it can "see" only the space directly in front of the the ^{22}Na sample and not the sample itself. Therefore it is optimal to place this tube perpendicular to the first scintillator and offset in front by a small distance. Furthermore, we shielded the setup via lead block to help avoid other sources of radiation in the room. The anode should then be connected via T junction to both channel 3 on the oscilloscope and the input of the second discriminator. This is quite similar to the setup of the first scintillator, however refer to the block diagram for clarification if needed. Similarly, connect the output of the discriminator channel to both channel 3 of the scope and the "STOP" input on the TAC. Now we can go ahead and calibrate the voltage threshold for this second discriminator channel. In a similar manner to the first discriminator, set up the threshold and windows to isolate the low amplitude, positronium annihilation band on the scope. Having properly configured the discriminator, you should have the bulk of your experimental set up complete. The range on the TAC should be set to 50ns and the multiplier to 10. We know the life span of positronium is expected to be on the order of a couple 100s of nanoseconds, therefore, by setting the TAC range to 500ns ($10 * 50ns$), we can isolate events that happen within the expected life span range and assure that we do not see other coincidental events.

Last, we will set up the TAC to output its pulse to the computer for data recording. Connect the 93 Ω output port on the TAC via T junction to both a second oscilloscope and the "ADC Input" port on the **Multichannel Analyzer (MCA)** and terminate the input with a 100 Ω terminator. This MCA provides an interface between the experimental setup and the computer, so that we may record the results in the GammaVision software on the PC. The MCA should be connected to the computer and interfaced with the GammaVision software. The GammaVision software should be set up in a fashion similar to the special relativity experiment. The oscilloscope should be set to trigger on a relatively low voltage and should be set to single trigger. The waveform may take a while to appear. The observation of both positronium formation and annihilation within the expected decay range is a relatively rare occurrence. We were seeing one event every 2-5 minutes. Given the newer, higher concentra-

tion ^{22}Na sample, this count rate should now be closer to 10 Hz. If you run the GammaVision software, you should be able to see data points collected into "bins" corresponding to the amplitude of the delay between the TAC start and stop signals (and therefore the lifespan of the positronium events).

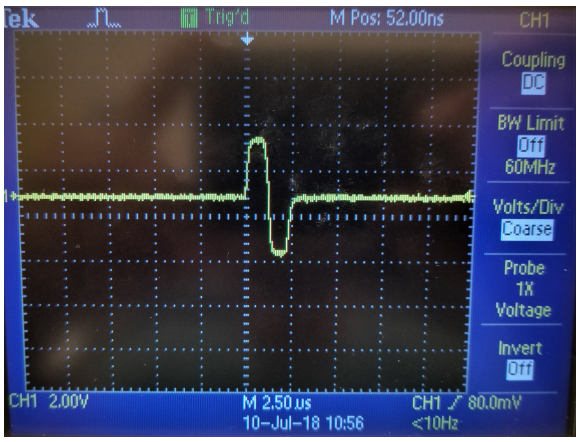

Figure 6. A photograph of the output pulse from the TAC on the second oscilloscope. The amplitude of this pulse reflects the decay rate of the positronium.

You are now almost ready to begin your data runs. After ascertaining that you are seeing pulses from the TAC on the oscilloscope and that the data is being recorded into GammaVision, the last instrumentation step is to calibrate the GammaVision software. In what seems to be a counter-intuitive step, remove the stop and start cables from the TAC. Instead, run coaxial cables from the "start" and "stop" ports on the Ortec Time Calibrator to the corresponding start and stop ports on the TAC. The time calibrator outputs start and stop pulses on a set interval, allowing you to proportionally relate the amplitude of the TAC signal and corresponding GammaVision bin with an exact time value. Set the range on the calibrator to 1.28μs. As our experiment is focused on events of the nanosecond variety, 1.28μs should be more than sufficient to accurately calibrate our setup. Now, set the period to 0.08μs and run the GammaVision software. You should be able to see data points appear in distinct vertical peaks as the run time increases. After letting the software gather data for a few minutes, you should be able to isolate the bins in which the peaks occur. The spacing between these bins can therefore be determined to be 0.08μs. Note the bin numbers in your lab notebook. Repeat this process with varying periods on the calibrator. After a few runs, you should be able to associate each bin on the GammaVision software with a precise time interval. After doing so, disconnect the calibrator from the setup and reconnect the discriminator cables to the TAC.

The final step is to evacuate the vacuum jar and pump nitrogen into the apparatus. Be sure to cycle the vacuum-nitrogen sequence a few times to make sure you have removed as much of the air in the apparatus as possible. After the cycling process, fill the vessel with nitrogen to a pressure slightly below ambient pressure. We have had good results at 70% atm. pressure. A fun later experiment to do might be to vary the pressure or gas in the vessel and see the decay changes in the positronium.

You are now ready for your data runs. This experiment should be run for as long as possible to acquire adequate data. Ask your professor if you are able to run the experiment overnight, or over the course of a weekend.

IV. DATA ANALYSIS AND DISCUSSION

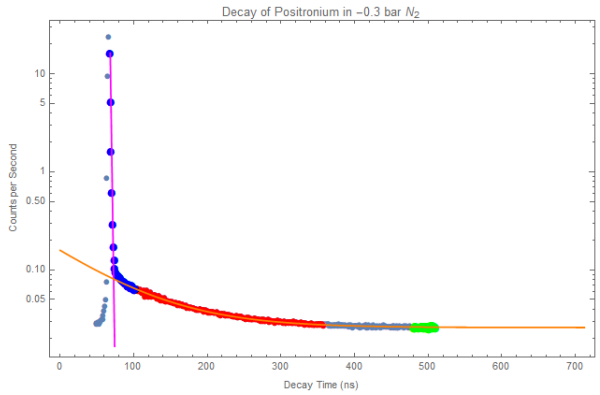

Figure 7. A log plot raw data from the GammaVision software: a histogram of counts versus bins corresponding to delay times. The red dots represent our Equation 3 fitted orthopositronium data while the orange represents our fit. The blue dots correspond to parapositronium, while the magenta line is an exponential fit for the parapositronium peak, allowing us to select red points greater than 50 lifetimes away from the parapositronium and therefore isolating the orthopositronium we care about. The green dots represent background data.

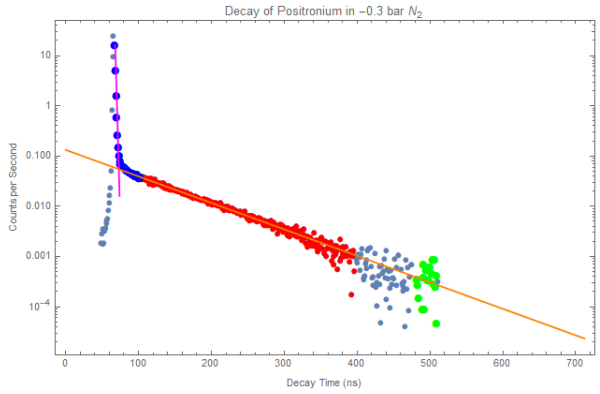

Figure 8. A log plot of number of events versus decay time. The data is the same as in Figure 7, however has been background subtracted.

The raw data from the GammaVision software should

resemble Figure 7. The peak in the data represents the most likely decay time for parapositronium, while the exponential regression after the peak represents the decay of orthopositronium. By fitting the tail of the parapositronium peak to Equation 3, you should be able to accurately predict the decay constant and half-life of orthopositronium in your gas and pressure (See Appendix A). Given the "pick-off" decay apparent due to the presence of an approximately 22% concentration of oxygen in the air, the decay constant obtained should be undoubtedly larger than that of the accepted value in vacuum (138.6 ns) if the experiment has any air in the apparatus.[13]

A. Improvements

You might want to play with the gas or pressure in the chamber. We have argon available for use in the lab and it would be an interesting comparison to see between air, argon, and nitrogen at the same or varying pressures. You might also play with moving the position of the upper scintillator up and down to find how high the positrons are penetrating by finding the point with the highest counts. There are a plethora of improvements and experiments to be performed and each will lead to this lab becoming more and more significant!

B. Acknowledgements

This experiment was designed and conducted with assistance and supervision from our TAs, Taha Dawoodbhoy, Matthew Durkee, and Robert Rosati, along with our professor, Dr. Greg O. Sitz; all of whom we would like to thank. Further thanks to Matt Staab for his help in improving the experiment from our initial run.

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APPENDIX A: MATHEMATICAL ANALYSIS OF DATA

There are a variety of different approaches to quantifying the separation of parapositronium, orthopositronium, and background coincidence events from the amalgam of data in Figure 7. The selection of the appropriate orthopositronium data makes a significant difference in the values of the decay constant and half life predicted by the exponential fit. There is really no obvious or purely quantitative way to choose only the orthopositronium data without some sort of subjective analysis, but I believe I have found a method which relatively minimizes subjectivity and provides a mathematical model for isolating the most valid orthopositronium decay data. This method is outlined as follows.

To begin with, I arbitrarily chose a finite amount of points in highest range of decay times as representing the background level of coincidence events in the experiment. For me, this was the last thirty data points in the usable data we gathered. This background data is represented in green in Figure 7 and Figure 8. Using this subset of data, I calculated the mean and standard deviation of the background noise. Moving forward, I have found two methods to work the best and provide some quantitative reasoning for choosing the points corresponding to orthopositronium decay data.

Method 1: Background Subtraction

For method one, I subtracted the mean of the background data from all the usable raw data shown in Figure 7. This yielded Figure 8, where the two distinct exponential decays corresponding to para- and ortho- decay are easily visible as converging lines on the logarithmic plot.

Using this background subtracted data, I then found the data point corresponding to the maximum, i.e, the parapositronium peak. I selected an arbitrary amount of points to the right of this peak to correspond to the decay of parapositronium, this time thirty-five points as noted in blue on Figure 8. This number, and the number of points selected as background noise, are the only parameters set subjectively by myself. The program I wrote then chooses the points best corresponding to orthopositronium in the manner described below:

I fit a simple exponential (with no constant baseline term) to the data chosen as parapositronium decay. This fit is shown in magenta on Figure 8. Using this fit, I was able to approximate the half life of parapositronium. I then used this half life calculation to place a lower limit on the data I chose for the orthopositronium fit, selecting only the data points at least fifty lifetimes away from the parapositronium peak, at which point I assume the parapositronium contribution to the decay curve is negligible.

To find the maximum cut off for the orthopositronium data, I cut off any trailing data within two standard deviations of the background level as calculated earlier. Combined with the minimum cut off above, I am left with the data shown in red on Figure 8. I then took a fit of Equation 3 with this truncated data (shown in orange on Figure 8) to calculate the value of the decay constant to reasonable and explainable degree of uncertainty.

Method 2: Adding a Background Constant

For method two, I do not subtract the background data, and proceed in the manner described in method one to isolate the orthopositronium data. When it comes to fitting the orthopositronium decay, however, I added a constant to the fit model of Equation 3 such that it becomes:

$$N(t) = N_0 e^{-\lambda t} + C \tag{5}$$

where C is a constant signifying the magnitude of the background coincidence events. This constant should have a value approximating the mean of the background calculated above. The end product of this method looks like Figure 7 where green points once again represent the background data, blue points represent parapositronium data, the magenta line represents the parapositronium decay, and red points represent the isolated orthopositronium data, which is subimposed on the orange fit line of Equation 5.

It should be noted that the two methods produce marginally different but similar results and, in my experience, method one seems to yield a more accurate prediction for the orthopositronium half-life. If you find a less “hand-wavy” approach to choosing the orthopositronium data, feel free to shoot me an email, I’d love to see your approach!